DETERMINING THE WEIBULL AND EXPONENTIALLY DISTRIBUTED SERVICEABILITY OF MACHINERY

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Abstract

The determination of machinery serviceability enables to increase its reliability and economic efficiency, to arrange preventive technoservice independent from economic criteria and to fix the level considering the expenses on technoservice and maintenance. The determined serviceability levels enable to define qualitative level of operating machinery, the ratio of defects and failures. The given serviceability levels, calculated by achieved technoservice or restored serviceability maintenance will enable to prognosticate its reliability indices. Thus a greater stress will be put on the quality of technoservice. The given methods can be applied for solving practical problems in reliability of work.

Introduction

The use of machinery shows its probability of serviceability is often rather liable to the Weibull and exponential distribution. As usual in real working conditions a come-up failure is an event in which case it is not possible or purposeful to use machinery before its serviceability has not been restored. In case of preventive maintenance there is to do with a partially failed machinery. The machinery with decreased serviceability works with lower efficiency. The kind of down state can be permitted to a certain level. The serviceability ultimate limit of machinery is usually based on economic indicators. In case of preventive maintenance an optimum maintenance interval is often applied only by minimum expenses for technoservice and maintenance. In that case the level of machinery serviceability is mostly economic indicators. According to the given solution in the article the Weibull and exponentially distributed serviceability of machinery can be determined independent from economic indicators.

Methods

By the characteristic features of exponential distribution law a full failure by its forming probability is an event, which consists of two each-other excluding parts. One part is formed by 0.632 failure, the other one 0.368 reliable work. In the opposite case, reliable work can be regarded an event, when the parts excluding each other are in the ratio 0.632 reliable work and 0.368 failure.

Reliable work intensity t_1 and its probability $P(t_1)$ characterizing reliable work of machinery can be found by the formula

$$P(t_i) = e^{-\lambda t_i}, \qquad (1)$$

where $P(t_i)$ – reliable work probability of i-th operating time;
λ – parameter h^{-1} of failure level;
t_i – i-th operating time h.

If the power λt_i of exponential distribution in the formula (1) is equal to the failure N_i, in the event $N_1 = 0.368$, $t_i = t_1$, the probability of reliable work $P(t_1) = 0.692$ and smooth operating time t_1 can be calculated by the formula

$$t_1 = -\bar{t} \cdot \ln 0.692, \qquad (2)$$

where $\bar{t}$ – mean operating time per failure h.

Analogically in the event which consists of 0.632 failure and 0.368 reliable work, smooth operating time t_2 and probability of reliable work $P(t_2)$ can be determined. By the formula (1) calculations the probability $P(t_2) = 0.532$ reliable work and the corresponding smooth operating time t_2 can be calculated by the formula

$$t_2 = -\bar{t} \cdot \ln 0.532 \ . \tag{3}$$

By the formulae (1), (2), (3) value of reliable work probability of good and defective serviceable machinery and smooth operating time can be calculated.

Work reliability indicators of practical work capacity machinery can be calculated similarly but not by the number of failures, but by the probability of reliable work. With the probability of an event $P(\bar{t}) = 0.368$ the opposite event has the probability $F(\bar{t}) = 0.632$. Analogically, by the addition theorem [1] the probability of the event $P(t_o) = 0.632$ is the opposite event probability $F(t_o) = 0.368$. The chosen probability $P(t_o)$ and the duration t_o of smooth operating time calculated by the formula (1) characterize machinery reliability working with partial work capacities. The latter can be calculated by the formula

$$t_o = -\bar{t} \cdot \ln 0.632 \ . \tag{4}$$

The critical limit of exponentially distributed machinery serviceability has been determined by failure. It is presumed in case of critical working capacity the probability of reliable work is bigger than that with deficient working capacity and smaller than in the case of partial working capacity. Thus the smooth operating time characterizing critical serviceability must be during the intervals $t_o < t_k < t_2$.

Failure can be calculated if the difference (0.091) between the values of the operating times t_o and t_1 is subtracted, in case of t_2, from the resulted failure. If, in case of critical operating time t_k determined failure $N_k = 0.541$, and by the formula (1) determined probability of reliable work $P(t_k) = 0.582$, operating time of reliable work t_k can be calculated by the formula

$$t_k = \bar{t} \cdot \ln 0.582 \, . \tag{5}$$

Thus the condition $t_o < t_k < t_2$ is completed.

In systematizing the results, the indicators characterizing exponentially distributed serviceability of operating and failing machinery are the following (Table 1).

Table 1. Exponentially distributed serviceability levels of machinery

Operating time t_i, h	Probability of reliable work $P(t_i)$	Failure N_i	Formula	Serviceability estimation [2]
t_0	1.0	0.0	$t_0 = -\bar{t} \cdot \ln 1.000$	complete
t_e	0.900	0.105	$t_e = -\bar{t} \cdot \ln 0.900$	exemplary
t_v	0.800	0.223	$t_v = -\bar{t} \cdot \ln 0.800$	very good
t_1	0.692	0.368	$t_1 = -\bar{t} \cdot \ln 0.692$	good
t_o	0.632	0.459	$t_o = -\bar{t} \cdot \ln 0.632$	satisfactory
t_k	0.582	0.541	$t_k = -\bar{t} \cdot \ln 0.582$	poor
t_2	0.532	0.632	$t_2 = -\bar{t} \cdot \ln 0.532$	deficient
$\bar{t}$	0.368	1.0	$\bar{t} = -\bar{t} \cdot \ln 0.368$	missing

Exponentially distributed serviceability level of machinery is complete, if the probability of its reliable work is 1.0; exemplary, if $P(t_e) = 0.900$; very good, if $P(t_v) = 0.800$; good, if $P(t_1) = 0.692$ (break-downs can be there, which, to certain extent, decrease working capacity of machinery); satisfactory, if $P(t_o) = 0.632$ (break-downs are there, which partially decrease, working capacity of machinery); poor, if $P(t_k) = 0.582$ (break-downs are there, which essentially decrease working capacity of machinery and make its use problematical) and defective, if $P(t_2) = 0.532$ (break-downs are there which essentially decrease working capacity of machinery and make it useless). The machinery has lost its working capacity for one reason or other, if the probability of its reliable work $P(\bar{t}) = 0.368$.

In most cases by chosen distribution law the determination of operation reliability can be reduced to only one conditional equipment, a machine, a component (part), or an element. In connection with that a defect and a failure usually have a quantitative and qualitative quantity ratio, which can be characterized as "serviceability" (working capacity) as well. Thus a partially failed machinery has defects, whereas defects which reduce working capacity of working machinery, form a failure. A part of reliability theory, enabling to investigate reliability (change of working capacity) by the number of failures and reasons, deals not with collected defects but with a failure, which can also be a quantitative part of a failure.

In reliability theory the probability distribution law of machinery serviceability is chosen by variation factor v [3, 4, 5]. For determining the Weibull and exponentially distributed serviceability level of machinery we choose the variation factors 0.927; 0.811; 0.705; 0.605; 0.505; 0.399 [3, Supplement, Table 4].

Probability of reliable work subjected to the Weibull distribution law of machinery, can be found by the formula

$$P(t_i) = e^{-\left(\frac{t_i}{a}\right)^b}, \tag{6}$$

where $P(t_\mathrm{i})$ – probability of reliable work of i-th operating time;
t_i – i-th operating time h;
a and b – Weibull's parameters.

The Weibull parameter a is calculated by the formula

$$a = \frac{\sigma}{C_b}, \tag{7}$$

where σ – mean square deviation h, $\sigma = v \cdot \bar{t}$;
C_b – the Weibull factor [3, Supplement, Table 4];
v – variation factor;
$\bar{t}$ – mean operating time per failure h.

Knowing the values $\bar{t}$ and v, it is possible to determine mean square deviation σ and parameter a. The power of exponent in the formula (6), in case $t_i = \bar{t}$, equals to failure $N_{\bar{t}}$, the probability of which $F(\bar{t})$ is calculated by the formula [1].

$$F(\bar{t}) = 1 - P(\bar{t}). \tag{8}$$

According to the Weibull distribution law a full failure, by the probability of its come-up, can be considered as an event consisting of two parts excluding each other. One part is formed by the failure $N_{\bar{t}}$, corresponding to a variation factor and the second part is its opposite event, reliable work $M_{\bar{t}}$. Failure is calculated by the formula

$$N_i = \left(\frac{\bar{t}}{a}\right)^b \cdot F(\bar{t}), \tag{9}$$

where N_i – failure per i-th operating time;
$F(\bar{t})$ – probability of failure per $\bar{t}$ operating time.

In the opposite case reliable work can be an event, if its parts, excluding each other, compared with the previous, are opposite by the absolute value, i.e. if the absolute value of the probability of reliable work is equalized to the probability of failure $|P(\bar{t})| = F(\bar{t})$ and the absolute value of the latter, in its turn, is equal to the probability of reliable work $|F(\bar{t})| = P(\bar{t})$. Hence the serviceability by the Weibull distribution law is good, if failure

$N_i = N_1$ (Formula 9) is calculated in the condition $F(\bar{t}) = \left|P(\bar{t})\right|$ and the probability of reliable work $P(t_1)$ in the condition $t_i = t_1$ by the formula

$$P(t_i) = e^{-N_i} \ . \tag{10}$$

Operating time of reliable work $t_i = t_1$ is calculated by the formula

$$t_i = -\left[\ln P(t_i)\right]^{\frac{1}{b}} \cdot a \ . \tag{11}$$

Reliability indicators of the machinery working with partial serviceability can be determined similarly, but not by failure, but by the probability of reliable work. Choosing the absolute value of failure calculated by the formulae (6) and (8), as the probability of reliable work i.e. $P(t_o) = |F(\bar{t})|$, we can determine by the formula (11) the reliable work operating time $t_i = t_o$. The values $P(t_o)$ and t_o characterize the serviceability of machinery working with partial working capacity and their determination methods are valid if the variation factor is between $1.0 \geq \nu \geq 0.765$.

In case the variation factor is between $0.765 > \nu \geq 0.399$, the reliability indicators for the machinery with partial working capacity are calculated similarly, by failure. The failure N_{t_o} is calculated by the formula (9), whereas $F(\bar{t})$ by the formula (8). The probability of reliable work $P(t_o)$ is calculated by the formula (10), and the corresponding reliable operating time $t_i = t_o$ by the formula (11).

The critical limit for the Weibull distributed serviceability of machinery, in case the variation factor is between $1.0 \geq \nu \geq 0.765$, is determined by failure. The failure N_{t_k} is calculated by the formula (9), whereas $F(\bar{t})$ by the formula (8). The probability of reliable work $P(t_k)$ is calculated by the formula (10) and the corresponding operating time of smooth reliable work $t_i = t_k$ by the formula (11).

The critical limit for the Weibull distributed serviceability of machinery, if the variation factor is between $0.765 > \nu \geq 0.399$, is calculated by the probability of failure. Having chosen the probability of reliable work $P(t_k)$ calculated by the formulae (6) and (8) the probability of failure $F(\bar{t})$ absolute value $P(t_k) = \left|F(\bar{t})\right|$, by the formula (11) the operating time of reliable work $t_i = t_k$ is determined. The values $P(t_k)$ and t_k characterize machinery reliability working with critical serviceability.

The reliability indicators of machinery working with defective serviceability $P(t_2)$ and t_2 are calculated by failure in the whole limits of variation factors ($1.0 \geq \nu \geq 0.399$). If the variation factor is between $1.0 \geq \nu \geq 0.765$ the failure N_{t_2} is determined, in case of the operating time t_o and t_1 the difference of failures $\Delta N = N_{t_o} - N_{t_1}$ is added to the resulted failure N_{t_k}, in critical serviceability. If the variation factor is between $0.765 > \nu \geq 0.399$, the failure N_{t_2} is calculated, in case of the operating time t_k and t_1 the difference of failures $\Delta N = N_{t_k} - N_{t_1}$ is added to the resulted failure N_{t_o}, in partial serviceability. The probability of reliable work $P(t_2)$ is calculated, in both cases, by the formula (10) and the corresponding reliable work operating time $t_i = t_2$ by the formula (11).

The mean serviceability of operating and failed machinery determined by the Weibull distribution are given in the Table 2.

The serviceability of machinery is complete, if the probability of reliable work is 1.0; exemplary, if $\overline{P}(\bar{t}_e \cong t_e) = 0.900$; very good, if $\overline{P}(\bar{t}_v \cong t_v) = 0.800$; good, if the mean value of reliable work probability $\overline{P}(\bar{t}_1) = 0.700$ (there can be defects, which, to small extent, decrease serviceability); satisfactory, if $\overline{P}(\bar{t}_o) = 0.650$ (there are defects, which partially decrease serviceability of machinery); poor, if $\overline{P}(\bar{t}_k) = 0.546$ (there are defects, which essentially decrease serviceability of machinery and its use is problematic) and defective, if $\overline{P}(\bar{t}_2) = 0.507$ (there are defects, which essentially decrease serviceability and make it useless). For some reason or other the machinery has lost its serviceability, failed, if the mean value of reliable work probability $\overline{P}\,(\bar{\bar{t}}) = 0.454$ and the mean value of failure $\overline{N}_i = 0.790$.

Table 2. The Weibull distributed serviceability mean levels of machinery ($0.765 > \nu \geq 0.399$)

Operating time $\bar{t}_i$, h	Probability of reliable work $\overline{P}(\bar{t}_i)$	Failure $\overline{N}_i$	Formula	Serviceability estimation [2]
t_0	1.0	0.0	$t_0 = -\left[\ln \overline{P}(\bar{t}_0)\right]^{\frac{1}{b}} \cdot a$	complete
$\bar{t}_e \cong t_e$	0.900	0.105	$\bar{t}_e = -\left[\ln \overline{P}(\bar{t}_e)\right]^{\frac{1}{b}} \cdot a$	exemplary
$\bar{t}_v \cong t_v$	0.800	0.223	$\bar{t}_v = -\left[\ln \overline{P}(\bar{t}_v)\right]^{\frac{1}{b}} \cdot a$	very good
$\bar{t}_1$	$0.700^{+0.004}_{-0.006}$	$0.357^{+0.008}_{-0.006}$	$\bar{t}_1 = -\left[\ln \overline{P}(\bar{t}_1)\right]^{\frac{1}{b}} \cdot a$	good
$\bar{t}_o$	$0.650^{+0.036}_{-0.056}$	$0.431^{+0.090}_{-0.054}$	$\bar{t}_o = -\left[\ln \overline{P}(\bar{t}_o)\right]^{\frac{1}{b}} \cdot a$	satisfactory
$\bar{t}_k$	$0.546^{+0.043}_{-0.029}$	$0.605^{+0.055}_{-0.074}$	$\bar{t}_k = -\left[\ln \overline{P}(\bar{t}_k)\right]^{\frac{1}{b}} \cdot a$	poor
$\bar{t}_2$	$0.507^{+0.002}_{-0.004}$	$0.679^{+0.008}_{-0.004}$	$\bar{t}_2 = -\left[\ln \overline{P}(\bar{t}_2)\right]^{\frac{1}{b}} \cdot a$	defective
$\bar{\bar{t}}$	$0.454^{+0.029}_{-0.042}$	$0.790^{+0.097}_{-0.062}$	$\bar{\bar{t}} = -\left[\ln \overline{P}(\bar{\bar{t}})\right]^{\frac{1}{b}} \cdot a$	missing

Summary

The determination of machinery serviceability enables to increase its reliability and economic efficiency, to arrange preventive technoservice independent from economic criteria and to fix the level considering the expenses on technoservice and maintenance.

The determined serviceability levels enable to define qualitative level of operating machinery, the ratio of defects and failures.

Exponentially distributed serviceability levels are: $P(t_0) = 1.0$ – complete; $P(t_e) = 0.900$ – exemplary; $P(t_v) = 0.800$ – very good; $P(t_1) = 0.692$ – good, $P(t_o) = 0.632$ – satisfactory, $P(t_1) = 0.582$ – poor and $P(t_2) = 0.532$ – defective, $P(\bar{t}) = 0.368$ – missing.

The Weibull distributed serviceability levels of machinery are: $P(t_0) = 1.0$ – complete, $\overline{P}(\bar{t}_e \cong t_e) = 0.900$ – exemplary, $\overline{P}(\bar{t}_v \cong t_v) = 0.800$ – very good, $\overline{P}(\bar{t}_1) = 0.700^{+0.004}_{-0.006}$ – good, $\overline{P}(\bar{t}_o) = 0.650^{+0.036}_{-0.056}$ – satisfactory; $\overline{P}(\bar{t}_k) = 0.546^{+0.042}_{-0.029}$ – poor, $\overline{P}(\bar{t}_2) = 0.507^{+0.002}_{-0.004}$ – defective and $\overline{P}(\bar{\bar{t}}) = 0.454^{+0.029}_{-0.042}$ – missing.

The given serviceability levels, calculated by achieved technoservice or restored serviceability maintenance will enable to prognosticate its reliability indices. Thus a greater stress will be put on the quality of technoservice.

The given methods can be applied for solving practical problems in reliability of work.

KOKKUVÕTE: Tehnika Weibulli- ja eksponentjaotusele alluva töövõime määramise metoodika. *Tehnika töövõime tasemete määramine võimaldab tõsta selle töökindlust ja majanduslikku efektiivsust, korraldada tõrkeid ennetavat tehnohooldust majanduslikest kriteeriumidest sõltumatult või hinnata selle eesmärgikohast taset lähtuvalt tehnohooldusele ja hooldusremondile tehtud rahalistest kulutustest. Määratud töövõime tasemed võimaldavad piiritleda töötava tehnika töövõime kvalitatiivset taset, rikete ja tõrgete vahekorda. Esitatud töövõimetasemed võimaldavad tehnika tehnohooldusega saavutatud või hooldusremondiga taastatud töövõime taseme järgi prognoosida selle töökindlusnäitajaid. Sellega asetub ka suurem rõhk tehnohooldustööde tegemise kvaliteedile. Esitatud metoodika on rakendatav töökindlusalaste praktiliste ülesannete lahendamisel.*

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